MATH 111i WORKSHEET 9

Name: Solutions

> Work in groups to answer as many problems as you can. Ask questions if you get stuck. The numbers used on this worksheet may require a calculator. Keep in mind that numbers you will have on exams will be nice enough to do without a calculator.

1. Solve the following exponential equations. Simplify your answers, leaving them in terms of suitable logarithms and/or numbers.

(a) $2^{1-x} = 2^{2-3x}$

$1 - x = 2 - 3x$

$2x = 1$

Answer: $x = 1/2$

(b) $9^{x^2} = 9^{12-4x}$

$x^2 = 12 - 4x$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2)$

Answer: $x = -6$ or $x = 2$

(c) $6^{x^2-3x} = 6^{20+5x}$

$x^2 - 3x = 20 + 5x$

$x^2 - 8x - 20 = 0$

$(x+2)(x-10) = 0$

Answer: $x = -2$ or $x = 10$

(d) $9^x = 27^{2+x}$

$3^{2x} = 3^{3(2+x)}$

$2x = 3(2+x)$

$2x = 6 + 3x$

$-6 = x$

Answer: $x = -6$

(e) $3 = 14^{9-2x}$

$\log_{14}(3) = 9 - 2x$

$2x = 9 - \log_{14}(3)$

Answer: $x = \frac{9 - \log_{14}(3)}{2}$

(f) $8^{4x+1} = 1 = 8^0$

$4x + 1 = 0$

$4x = -1$

Answer: $x = -1/4$

(g) $10^{7x} = 3$

$7x = \log_{10}(3)$

Answer: $x = \frac{1}{7}\log_{10}(3)$

(h) $6^{2+x} = 8^{8+2x}$

$2 + x = (8+2x)\log_6(8)$

$x - 2x\log_6(8) = 8\log_6(8) - 2$

$x(1 - 2\log_6(8)) = 8\log_6(8) - 2$

Answer: $x = \frac{8\log_6(8) - 2}{1 - 2\log_6(8)}$

2. Given $f(x)$ and $g(x)$, find both $f(g(x))$ and $g(f(x))$.

(a) $f(x) = 2x + 9$, $g(x) = 2x - 1$

$f(g(x)) = 2g(x) + 9$
$= 2(2x-1)+9$

$g(f(x)) = 2f(x) - 1$
$= 2(2x+9) - 1$

Answer: $4x + 7$

Answer: $4x + 17$

(b) $f(x) = x^2 + 1$, $g(x) = 6 - 4x$

$f(g(x)) = g(x)^2 + 1$
$= (6-4x)^2 + 1$

$g(f(x)) = 6 - 4f(x)$
$= 6 - 4(x^2+1)$

Answer: $16x^2 - 48x + 37$

Answer: $2 - 4x^2$

(c) $f(x) = 2x^2 + 9$, $g(x) = 1 - 2x - x^2$

$f(g(x)) = 2g(x)^2 + 9$
$= 2(1-2x-x^2)^2 + 9$
$= 2[1 - 2x - x^2 - 2x + 4x^2 + 2x^3 - x^2 + 2x^3 + x^4] + 9$

$g(f(x)) = 1 - 2f(x) - f(x)^2$
$= 1 - 2(2x^2+9) - (2x^2+9)^2$

Answer: $11 - 8x + 4x^2 + 8x^3 + 2x^4$

Answer: $-4x^2 - 40x^2 - 98$

(d) $f(x) = 9x^2 + 10x + 12$, $g(x) = 2$

$f(g(x)) = 9g(x)^2 + 10g(x) + 12$
$= 9 \cdot 2^2 + 10 \cdot 2 + 12$

$g(f(x)) = 2$

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Answer: 2

(e) $f(x) = x + 1$, $g(x) = \frac{2}{x-3}$

$f(g(x)) = g(x) + 1 = \frac{2}{x-3} + 1$

$g(f(x)) = \frac{2}{f(x)-3} = \frac{2}{(x+1)-3}$

Answer: $\frac{x-1}{x-3}$

Answer: $\frac{2}{x-2}$

(f) $f(x) = \frac{1}{2}x - 3$, $g(x) = 2x + 6$

$f(g(x)) = \frac{1}{2}g(x) - 3$
$= \frac{1}{2}(2x+6) - 3$

$g(f(x)) = 2f(x) + 6$
$= 2(\frac{1}{2}x - 3) + 6$

Answer: x

Answer: x

Cont.

3. Given $f(x)$ and $g(x)$, find both $f(g(x))$ and $g(f(x))$.

(a) $f(x) = 10 \cdot 4^x$, $g(x) = \log_4\left(\frac{x}{10}\right)$

$f(g(x)) = 10 \cdot 4^{g(x)}$
$= 10 \cdot 4^{\log_4(x/10)}$
$= 10 \cdot x/10$

$g(f(x)) = \log_4\left(\frac{f(x)}{10}\right)$
$= \log_4(4^x)$

Answer: x

Answer: x

(d) $f(x) = 3x + 5$, $g(x) = \ln(x)$

$f(g(x)) = 3g(x) + 5$
$= 3\ln(x) + 5$

$g(f(x)) = \ln(f(x))$
$= \ln(3x+5)$

Answer: $3\ln(x)+5$

Answer: $\ln(3x+5)$

(b) $f(x) = 2x - 4$, $g(x) = 10^x$

$f(g(x)) = 2g(x) - 4$
$= 2 \cdot 10^x - 4$

$g(f(x)) = 10^{f(x)}$
$= 10^{2x-4}$

Answer: $2 \cdot 10^x - 4$

Answer: 10^{2x-4}

(e) $f(x) = \log_2(x)$, $g(x) = x^4 + 1$

$f(g(x)) = \log_2(g(x))$
$= \log_2(x^4+1)$

$g(f(x)) = f(x)^4 + 1$
$= \log_2(x)^2 + 1$

Answer: $\log_2(x^4+1)$

Answer: $\log_2(x)^2+1$

(c) $f(x) = e^x$, $g(x) = x + 3$

$f(g(x)) = e^{g(x)}$
$= e^{3+x}$

$g(f(x)) = f(x) + 3$
$= e^x + 3$

Answer: e^{3+x}

Answer: $e^x + 3$

(f) $f(x) = \ln(x^2 - 1)$, $g(x) = e^{2x}$

$f(g(x)) = \ln(f(x)^2 - 1)$
$= \ln(e^{2x} - 1)$

$g(f(x)) = e^{2f(x)}$
$= e^{2\ln(x^2-1)}$
$= e^{\ln((x^2-1)^2)}$

Answer: $\ln(e^{2x}-1)$

Answer: $(x^2-1)^2$

Cont.

4. Given $f(x)$ and $g(x)$, determine if they are inverse of each other.

(a) $f(x) = \frac{3-x}{4}$, $g(x) = 3 - 4x$

$$f(g(x)) = \frac{3-g(x)}{4} = \frac{3-(3-4x)}{4} = x$$

$$g(f(x)) = 3 - 4f(x) = 3 - 4\cdot\frac{3-x}{4} = x$$

Answer: Yes

(b) $f(x) = \frac{1}{x-4}$, $g(x) = x - 4$

$$f(g(x)) = \frac{1}{g(x)-4} = \frac{1}{x-4-4} = \frac{1}{x-8}$$

Answer: No

(c) $f(x) = x^3 + 1$, $g(x) = \sqrt[3]{x-1}$

$$f(g(x)) = g(x)^3 + 1 = (\sqrt[3]{x-1})^3 + 1 = x - 1 + 1 = x$$

$$g(f(x)) = \sqrt[3]{f(x)-1} = \sqrt[3]{x^3+1-1} = x$$

Answer: Yes

(d) $f(x) = \frac{1}{x-1}$, $g(x) = \frac{1}{x} + 1$

$$f(g(x)) = \frac{1}{g(x)-1} = \frac{1}{\frac{1}{x}+1-1} = \frac{1}{1/x} = x$$

$$g(f(x)) = \frac{1}{f(x)} + 1 = \frac{1}{\frac{1}{x-1}} + 1 = x - 1 + 1 = x$$

Answer: Yes

(e) $f(x) = \frac{1+x}{x}$, $g(x) = \frac{x}{1+x}$

$$f(g(x)) = \frac{1+g(x)}{g(x)} = \frac{1+\frac{x}{1+x}}{\frac{x}{1+x}} = \frac{\frac{1+x+x}{1+x}}{\frac{x}{1+x}} = \frac{1+2x}{x}$$

Answer: No

(f) $f(x) = \log_5(x^2)$, $g(x) = 5^{x/2}$

$$f(g(x)) = \log_5(g(x)^2) = \log_5(5^x) = x$$

$$g(f(x)) = 5^{f(x)/2} = 5^{\frac{1}{2}\log_5(x^2)} = 5^{\log_5(x)} = x$$

Answer: Yes

(g) $f(x) = \ln(x-3)$, $g(x) = e^x + 3$

$$f(g(x)) = \ln(g(x)-3) = \ln(e^x+3-3) = \ln(e^x) = x$$

$$g(f(x)) = e^{f(x)} + 3 = e^{\ln(x-3)} + 3 = x - 3 + 3 = x$$

Answer: Yes

(h) $f(x) = \frac{1}{3}\ln(2x)$, $g(x) = 3e^x$

$$f(g(x)) = \frac{1}{3}\ln(2g(x)) = \frac{1}{3}\ln(6e^x)$$

Answer: No

Cont.

5. Given $f(x)$, find $f^{-1}(x)$.

(a) $f(x) = 4x + 7$

$y = 4x+7$

$y-7 = 4x$

$\frac{y-7}{4} = x$

Answer: $f^{-1}(x) = \frac{x-7}{4}$

(b) $f(x) = 3 - 5x$

$y = 3-5x$

$5x = 3-y$

$x = \frac{3-y}{5}$

Answer: $f^{-1}(x) = \frac{3-x}{5}$

(c) $f(x) = \frac{x}{2}$

$y = \frac{x}{2}$

$2y = x$

Answer: $f^{-1}(x) = 2x$

(d) $f(x) = x^3 - 4$

$y = x^3 - 4$

$y+4 = x^3$

$\sqrt[3]{y+4} = x$

Answer: $f^{-1}(x) = (y+4)^{1/3}$

(e) $f(x) = \sqrt[3]{x+2}$

$y = \sqrt[3]{x+2}$

$y^3 = x+2$

$y^3 - 2 = x$

Answer: $f^{-1}(x) = x^3 - 2$

(f) $f(x) = 12x - 2$

$y = 12x-2$

$y+2 = 12x$

$\frac{y+2}{12} = x$

Answer: $f^{-1}(x) = \frac{x+2}{12}$

(g) $f(x) = \frac{1+x}{3-x}$

$y = \frac{1+x}{3-x}$

$(3-x)y = 1+x$

$3y - xy = 1+x$

$3y-1 = x+xy$

$3y-1 = x(1+y)$

$3y-1 = x(1+y)$

$x = \frac{3y-1}{1+y}$

Answer: $f^{-1}(x) = \frac{3x-1}{1+x}$

(h) $f(x) = \frac{x-2}{x+2}$

$y = \frac{x-2}{x+2}$

$y(x+2) = x-2$

$xy + 2y = x-2$

$2+2y = x-xy$

$2+2y = x(1-y)$

$x = \frac{2+2y}{1-y}$

Answer: $f^{-1}(x) = \frac{2+2y}{1-y}$

6. Given $f(x)$, find $f^{-1}(x)$.

(a) $f(x) = \log_2(x+1)$

$y = \log_2(x+1)$

$2^y = x+1$

$2^y - 1 = x$

Answer: $f^{-1}(x) = 2^x - 1$

(b) $f(x) = 10^{3x}$

$y = 10^{3x}$

$\log_{10}(y) = 3x$

$\frac{1}{3}\log_{10}(y) = x$

Answer: $f^{-1}(x) = \log_{10}(\sqrt[3]{y})$

(c) $f(x) = e^{0.5x}$

$y = e^{\frac{1}{2}x}$

$\ln(y) = \frac{1}{2}x$

$2\ln(y) = x$

Answer: $f^{-1}(x) = \ln(x^2)$

(d) $f(x) = \log_3(2x)$

$y = \log_3(2x)$

$3^y = 2x$

$\frac{1}{2}3^y = x$

Answer: $f^{-1}(x) = \frac{1}{2}3^x$

(e) $f(x) = \ln(x-3)$

$y = \ln(x-3)$

$e^y = x-3$

$e^y + 3 = x$

Answer: $f^{-1}(x) = e^x + 3$

(f) $f(x) = 2^{x^3}$

$y = 2^{x^3}$

$\log_2(y) = x^3$

$\sqrt[3]{\log_2(y)} = x$

Answer: $f^{-1}(x) = \log_2(y)^{1/3}$

(g) $f(x) = \log_4(x^3 - 1)$

$y = \log_4(x^3 - 1)$

$4^y = x^3 - 1$

$4^y + 1 = x^3$

$\sqrt[3]{4^y + 1} = x$

Answer: $f^{-1}(x) = (4^x + 1)^{1/3}$

(h) $f(x) = e^{1 + \sqrt[3]{3x+4}}$

$y = e^{1 + \sqrt[3]{3x+4}}$

$\ln(y) = 1 + \sqrt[3]{3x+4}$

$\ln(y) - 1 = (3x+4)^{1/3}$

$(\ln(y) - 1)^3 = 3x + 4$

Answer: $f^{-1}(x) = \frac{(\ln(x)-1)^3 - 4}{3}$

$(\ln(y) - 1)^3 - 4 = 3x$

$\frac{(\ln(y) - 1)^3 - 4}{3} = x$

The End.